PSAT SAMPLE
Math-No Calculator

A) If $a^{-\frac{1}{2}} = x$, where $a > 0$ and $x > 0$, which of the following equations gives a in terms of x?

Select an Answer

(A) $a = \frac{1}{\sqrt{x}}$

(B) $a = \frac{1}{x^2}$

(C) $a = \sqrt{x}$

(D) $a = -x^2$

B) If $a^2 + 14a = 51$ and $a > 0$, what is the value of $a + 7$?

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Choice B is correct. There are multiple ways to approach this problem, but all require an understanding of the properties of exponents. The student may rewrite the equation as $\frac{1}{\sqrt{a}} = x$ and then proceed to solve for a first by squaring both sides, which gives $\frac{1}{a} = x^2$, and then multiplying both sides by a to find $1 = ax^2$. Finally, dividing both sides by x^2 isolates the desired variable.

Choice A is not the correct answer. This answer may result from a misunderstanding of the properties of exponents. The student may understand that a negative exponent can be translated to a fraction but misapply the fractional exponent.

Choice C is not the correct answer. This answer may result from a misunderstanding of the properties of exponents. A student may recognize that an exponent of $\frac{1}{2}$ is the same as the square root but misapply this information.

fractional exponent.

Choice C is not the correct answer. This answer may result from a misunderstanding of the properties of exponents. A student may recognize that an exponent of $\frac{1}{2}$ is the same as the square root but misapply this information.

Choice D is not the correct answer. This answer may result from a misunderstanding of the properties of exponents. The student may recognize that the fractional exponent on a is the same as the square root, and that therefore a can be isolated by squaring both sides. However, the student may not understand how the negative exponent affects the base.

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There is more than one way to solve this problem. A student can apply standard techniques by rewriting the equation $a^2 + 14a = 51$ as $a^2 + 14a - 51 = 0$ and then factoring. Since the coefficient of a is 14 and the constant term is -51, factoring $a^2 + 14a - 51 = 0$ requires writing 51 as the product of two numbers that differ by 14. This is $51 = (3)(17)$, which gives the factorization $a^2 + 14a - 51 = (a + 17)(a - 3) = 0$. The possible values of a are $a = -17$ and $a = 3$. Since it is given that $a > 0$, it must be true that $a = 3$. Thus, the value of $a + 7$ is $3 + 7 = 10$.

A student could also use the quadratic formula to find the possible values of a:

$$a = \frac{-14 \pm \sqrt{14^2 - 4(1)(-51)}}{2(1)}$$
$$= \frac{-14 \pm \sqrt{196 - (-204)}}{2}$$
$$= \frac{-14 \pm \sqrt{400}}{2}$$
$$= \frac{-14 \pm 20}{2}$$

The possible values of a are $a = \frac{-14 - 20}{2} = -17$ and $a = \frac{-14 + 20}{2} = 3$. Again, since it is given that $a > 0$, it must be true that $a = 3$. Thus, the value of $a + 7$ is $3 + 7 = 10$.

There is another way to solve this problem that will reward the student who recognizes that adding 49 to both sides of the equation yields $a^2 + 14a + 49 = 51 + 49$, or rather $(a + 7)^2 = 100$, which has a perfect square on each side. Since $a > 0$, the solution $a + 7 = 10$ is evident.

Question Difficulty: EASY

Objective:
Students must solve a quadratic equation.